



Rossmoyne Senior High School

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(5 marks)

The table below shows the duration and immediate predecessors for all the activities required to complete a project.

Activity	U	V	W	X	Y	Z
Duration (minutes)	17	45	21	11	13	32
Immediate predecessors	–	–	U	V	V, W	X, Y

(a) Construct a project network to show all the above information.

(3 marks)

do not penalise for (o) on dummy

Solution	
<pre> graph LR 0((0)) -- "U (17)" --> 17((17)) 0 -- "V (45)" --> 45((45)) 17 -- "W (21)" --> 45 45 -- "X (11)" --> 58((58)) 45 -- "Y (13)" --> 58 58 -- "Z (32)" --> 90((90)) 45 -.-> 45 </pre>	
Specific behaviours	
✓ network with correct precedence relations	✓
✓ uses dummy edge correctly	✓
✓ adds direction and labels to edges	✓

(b) Determine the minimum completion time for the project and list, in order, the activities that lie on the critical path. (2 marks)

Solution	
Minimum completion time is 90 minutes.	✓
Activities on critical path: V, Y, Z.	✓
Specific behaviours	
✓ minimum completion time	
✓ ordered list of activities on critical path	

(f.t) must show working

5

Question 10

(7 marks)

A builder bought a scissor lift for \$38 000 and for accounting purposes will depreciate its value by 15% annually.

- (a) Show that the value of the lift after one year will be \$32 300. (1 mark)

Solution
$\$38\,000 \times 0.85 = \$32\,300$ ✓
Specific behaviours
✓ clearly shows any correct method

- (b) Write a recursive rule to calculate the value of the lift, T_n in dollars, after n years. (2 marks)

Solution
$T_{n+1} = 0.85 \times T_n, \quad T_0 = 38\,000$ ✓
Specific behaviours
✓ recursive part ✓ initial term using T_0

- (c) Calculate the value of the lift after 4 years. (1 mark)

Solution
$T_4 = \$19\,836.24$ ✓
Specific behaviours
✓ correct value (or rounded to nearest \$)

(F.T.)
 $T_{n+1} = T_n - 5700$
 $T_0 = 38000$
 $\therefore T_4 = 20900$

- (d) Deduce the n^{th} term rule for the value of the lift after n years. (1 mark)

Solution
$T_n = 38\,000(0.85)^n$ ✓
Specific behaviours
✓ correct rule

or $T_n = 32300 \times 0.85^{n-1}$

- (e) Determine the least number of years that must pass for the lift to depreciate by more than 80% of its original value. (2 marks)

Solution
20% of value will remain, so value will be \$7600. ✓ $T_9 = 8801, \quad T_{10} = 7481.$
Least number of years is 10. ✓
Specific behaviours
✓ indicates required value ✓ correct number of years

ans only ok (F.T.) 7 years

Question 11

(9 marks)

The eye diameter and body length of eight tropical fish of the same species are shown in the table below.

Eye diameter, D mm	8.7	6.9	10.4	8.3	8.0	7.7	9.8	9.1
Body length, L cm	6.7	11.5	3.8	8.2	9.1	10.3	5.3	7.3

- (a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (1 mark)

Solution
The association is strong and negative. ✓
Specific behaviours
✓ states strong and negative

- (b) Determine the coefficient of determination for the linear association and interpret its value in context. (2 marks)

Solution
$r^2 = 0.967$ ✓
97% of variation in the body length can be explained by the variation in eye diameter. ✓
Specific behaviours
✓ r^2 , to at least 2 decimal places
✓ correct interpretation of coefficient

- (c) Determine the equation of the least-squares line to predict L from D and interpret the slope of the line in context. (3 marks)

Solution
$L = -2.2D + 26.72$ ✓ ✓ <i>variables</i>
The slope means that for every 1 mm increase in eye diameter, the length of a fish is expected to decrease by 2.2 cm. ✓
Specific behaviours
✓ correct values in equation
✓ with given variables
✓ indicates as one variable increases, the other decreases

- (d) Predict the body length of another tropical fish of the same species that has an eye diameter of 8.1 cm and give two reasons that support the validity of this prediction. (3 marks)

Solution
$L(8.1) = 8.9$ cm ✓
Prediction is valid because correlation is strong, and it does not involve extrapolation. ✓
Specific behaviours
✓ calculates diameter
✓ states strong correlation
✓ states interpolation or no extrapolation

9

Question 12

(8 marks)

A researcher observed a large number of mice, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

Mice	Left-pawed	Right-pawed
Male	39	63
Female	23	60

The researcher is interested in whether there is an association between the variables.

- (a) Name one of the variables in the study and classify it as numerical or categorical. (2 marks)

Solution
Name: Gender or Paw. ✓
Classifies as Categorical. ✓
Specific behaviours
✓ names one variable
✓ classifies variable

- (b) Determine the percentage of left-pawed mice that were male. (2 marks)

Solution
$\frac{39}{39 + 23} = \frac{39}{62} = 62.9\%$ ✓
Specific behaviours
✓ indicates correct total for category
✓ correct percentage (whole number ok)

*accept 63%
ans only ok*

- (c) Use the above data to complete the following table so that it shows column percentages rounded to the nearest whole number. (2 marks)

Mice (%)	Left-pawed	Right-pawed
Male	63	51
Female	37	49

(100) ✓ (100)

Solution
See table
Specific behaviours
✓ both columns add to 100
✓ correct table

- (d) Explain whether the percentaged table suggests the presence of an association between the variables. (2 marks)

Solution
An association is evident between the variables as the pairs of percentages in each row (i.e., for each gender) are quite different.
Specific behaviours
✓ states association ✓
✓ explanation using different percentages ✓

8

Question 13

(7 marks)

A reducing balance loan is defined by the recurrence relation $T_{n+1} = k \times T_n - m$, $T_1 = p$, where T_n is the balance of the loan in dollars at the start of month n . The relation was used to create the following spreadsheet.

Month n	Balance of loan at start of month n	Monthly interest	Monthly repayment	Loan balance carried forward
1	12 500.00	92.50	395.00	12 197.50
2	12 197.50	90.26	395.00	11 892.76
3	11 892.76	88.01	395.00	Q
4		R		S

- (a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

Solution
$92.50 \div 12500 = 0.0074$ ✓
$0.0074 \times 12 \times 100 = 8.88\% \text{ pa}$ ✓
Specific behaviours
✓ calculates monthly rate
✓ correct annual rate as percentage

ans only ok
must be %

- (b) State the value of each of the constants k, m and p in the recurrence relation. (2 marks)

Solution
$k = 1.0074$ ✓
$m = 395, \quad p = 12\,500$ ✓
Specific behaviours
✓ value of k
✓ value of m and value of p

- (c) Determine the value of Q , the value of R and the value of S shown in the spreadsheet. (3 marks)

Solution
$Q = 11892.76 + 88.01 - 395.00 = \$11\,585.77$ ✓
$R = 11\,585.77 \times 0.0074 = \85.73 ✓
$S = 11\,585.77 + 85.73 - 395.00 = \$11\,276.50$ ✓
Specific behaviours
✓ value of Q
✓ value of R
✓ value of S

(f.t)

Question 14

(9 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

Week	Day (<i>n</i>)		New Clients	Weekly mean	Percentage of weekly mean
1	Mon	1	49	A	98.4
	Tue	2	53		106.4
	Wed	3	53		106.4
	Thu	4	49		B
	Fri	5	45		90.4
2	Mon	6	58	60.6	95.7
	Tue	7	66		108.9
	Wed	8	62		102.3
	Thu	9	63		104.0
	Fri	10	54		89.1
3	Mon	11	68	70.2	96.9
	Tue	12	C		106.8
	Wed	13	72		102.6
	Thu	14	71		101.1
	Fri	15	65		92.6

(a) Calculate the value of **A**, the value of **B** and the value of **C** in the table.

(3 marks)

Solution	
$A = (49 + 53 + 53 + 49 + 45) \div 5 = 49.8$	✓
$B = 49 \div 49.8 = 98.4\%$	✓
$C = 70.2 \times 106.8\% = 75$	✓
Specific behaviours	
✓ value of <i>A</i>	
✓ value of <i>B</i>	
✓ value of <i>C</i>	

3

(b) Determine the seasonal index for Mondays.

(2 marks)

Solution
$I_{MON} = (98.4 + 95.7 + 96.9) \div 3$ $= 291 \div 3$ $= 97\% \quad \text{or } 0.97 \quad \checkmark$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct indices ✓ ✓ calculates index as percent or decimal

ans
only
ok

(c) The least-squares line to predict the deseasonalised number of new clients c from the day is $c = 1.83n + 45.6$. Stating any assumptions made, determine the best estimate for the number of new clients expected on Friday of Week 4. (4 marks)

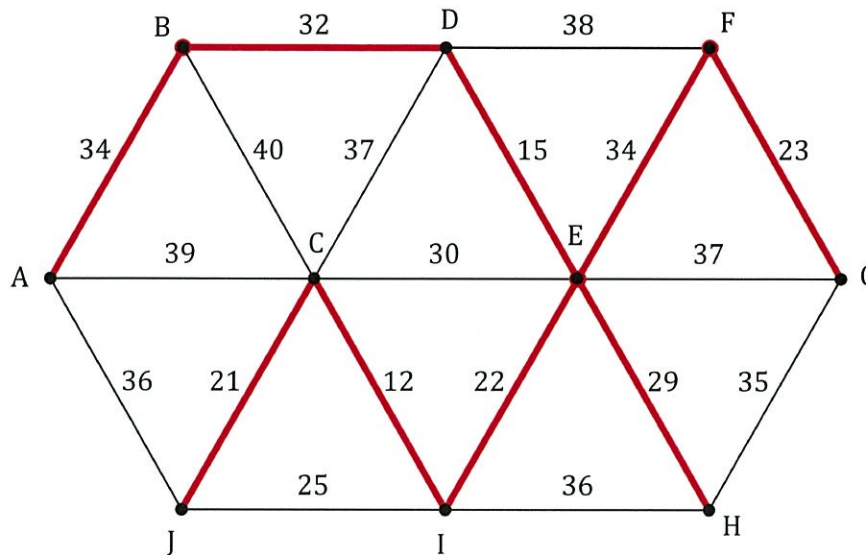
Solution
Friday of Week 4 is day 20. ✓
$c = 1.83(20) + 45.6$ $= 82.2$
$I_{FRI} = (90.4 + 89.1 + 92.6) \div 3$ $= 272.1 \div 3$ $= 90.7\% \quad \checkmark$
$C = 82.2 \times 0.907$ $= 74.6$ $\approx 75 \quad \checkmark$
Expect 75 new clients, assuming that the existing trend and seasonality continues into the future. ✓
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct day to calculate deseasonalised number ✓ calculates seasonal index for Friday ✓ uses index to calculate whole number estimate ✓ states assumption

6

Question 15

(8 marks)

Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.



- (a) Clearly identify the minimum spanning tree on the graph above. (3 marks)

Solution
See graph
Specific behaviours
✓ any tree; ✓ any spanning tree; ✓ minimum spanning tree

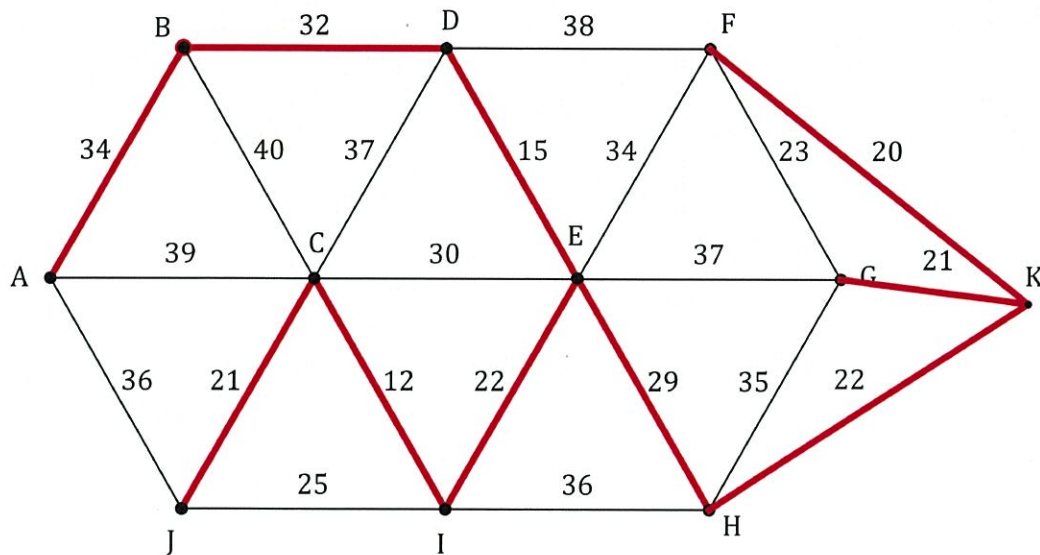
- (b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost \$18.50. (2 marks)

Solution
Sum of edges: $L = 34 + 32 + 15 + 34 + 23 + 21 + 12 + 22 + 29 = 222 \text{ m}$ ✓
Total cost: $C = 222 \times 18.50 = \$4107$ ✓
Specific behaviours
✓ sum of edges ✓ calculates cost

(f.t)
ans only
ok

5

- (c) Explain how your answer to part (b) will change if smoke alarm *K* is added to the system with cable lengths of 20, 21 and 22 metres to alarms *F*, *G* and *H* respectively. (A copy of the graph from the previous page is shown below if you wish to use it.) (3 marks)



Solution	
The minimum spanning tree changes so that edges <i>EF</i> and <i>FG</i> no longer used but edges <i>FK</i> , <i>GK</i> and <i>HK</i> are now included.	✓
The edge sum increases by 6 m to 228 m.	✓
Total cost increases by \$111 to \$4218.	✓
Specific behaviours	
✓ correctly shows new minimum spanning edges or states changes to edges	
✓ calculates increase in (or new) sum of edges	(f-t)
✓ calculates increase in (or new) cost	

Question 16

(7 marks)

Loans are offered by lender A at a rate of 10.32% per annum compounded quarterly and from lender B at a rate of 10.22% per annum compounded daily.

- (a) Calculate the total amount that must be repaid if \$1500 is borrowed from lender A for nine months. (2 marks)

or
 $A = 1500 \times 1.0258^3$
 $A = 1500 \times 1.0794$

Ans only
ok

Solution	
$A = 1500 \left(1 + \frac{10.32}{4 \times 100}\right)^3$ $A = 1619.12$ Hence \$1619.12 must be repaid.	Financial calculator $N = 3, I\% = 10.32, PV = -1500$ $PMT = 0, P/Y = 4, C/Y = 4$ Solve for $FV = 1619.12$ Hence \$1619.12 must be repaid.
Specific behaviours	
✓ shows compound interest formula or values used in financial calculator ✓ correct amount	

- (b) Calculate the total interest that will be charged on a loan of \$90 000 from lender B for 30 days. (2 marks)

Solution	
$A = 90\,000 \left(1 + \frac{10.22}{365 \times 100}\right)^{30}$ $A = 90\,759.08$ Hence \$759.08 interest charged.	Financial calculator $N = 30, I\% = 10.22, PV = -90000$ $PMT = 0, P/Y = 365, C/Y = 365$ Solve for $FV = 90\,759.08$ Hence \$759.08 interest charged.
Specific behaviours	
✓ shows compound interest formula or values used in financial calculator ✓ correct amount of interest	

- (c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

Solution
Lender A: $N = 4, APR = 10.32, EFF = 10.7263.$ ✓
Lender B: $N = 365, APR = 10.22, EFF = 10.7589.$ ✓
Hence interest will be minimised by choosing lender A (as their effective interest rate is less than that of lender B). ✓
Specific behaviours
✓ effective interest rate for A ✓ effective interest rate for B ✓ uses effective rates to choose lender

ans only
1 mark

Question 17

(8 marks)

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by $S_{n+1} = 0.95S_n + 950$, $S_0 = 1000$, where S_n is the concentration, in parts per million, after saltwater has been flowing into the tank for n minutes.

(a) Determine S_5 .

Solution
$S_5 = 5072$ ppm ✓
Specific behaviours
✓ correct value

accept
5071.94

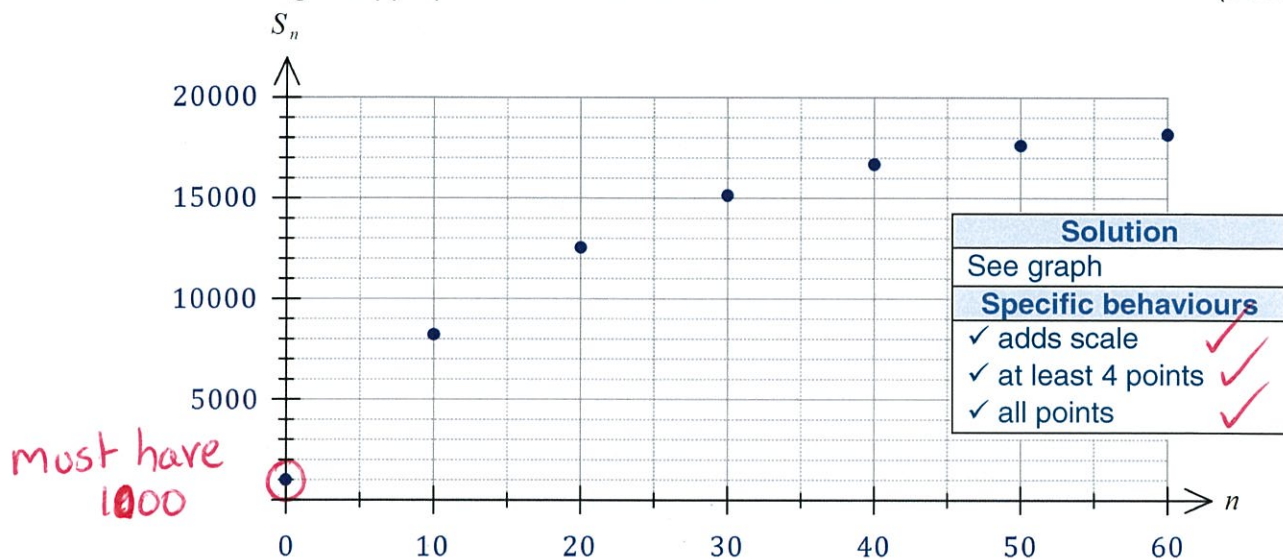
(1 mark)

(b) Determine the value of n for S_n to first exceed 10 000 ppm.

Solution
$n = 14$ ✓
Specific behaviours
✓ correct value

(1 mark)

(c) Plot points to show the salt concentration at 10 minute intervals on the axes below, after first adding an appropriate scale to the vertical axis. (3 marks)



(d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state. (1 mark)

Solution
Describes 'levelling off' or 'increasing at a decreasing rate', etc. ✓
Specific behaviours
✓ reasonable description

(e) Determine, with justification, the steady-state salt concentration. (2 marks)

Solution
$S = 0.95S + 950 \Rightarrow S = 19\ 000$ ppm ✓
Specific behaviours
✓ equation, or explanation using term(s) with large n
✓ correct concentration

ans only ok

8

Question 18

(8 marks)

A fund with a balance of \$525 000 is used to create an annuity, from which regular withdrawals of \$12 500 are to be made at the end of each quarter. Interest at a rate of 7.6% per annum is added to the fund quarterly, just before each withdrawal.

- (a) Use one or more calculations to show that the balance of the fund after one withdrawal is \$522 475. (2 marks)

or
 $1.019 (525000)$
 -12500
 $= 522475$

Solution	
$525\,000 \times (0.076 \div 4) = 9975$	$7.6 \div 4 = 1.9$
$525\,000 + 9975 - 12\,500 = 522\,475$	$525\,000 \times 1.019 - 12\,500 = 522\,475$
Specific behaviours	
<ul style="list-style-type: none"> ✓ indicates adjustment for interest for one quarter ✓ calculation: interest less withdrawal or recursive type 	

must have calculations

- (b) Write a recurrence relation to calculate the balance T_n after the n^{th} withdrawal. (2 marks)

Solution
$T_{n+1} = 1.019 \times T_n - 12\,500, \quad T_0 = 525\,000.$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct recursive part ✓ correct statement for T_0

- (c) Determine the total interest earned by the fund during its first year of operation. (2 marks)

Solution
$T_4 = 514\,608.49$
$514\,608.49 + 4 \times 12\,500 - 525\,000 = 39\,608.49$
Total interest is \$39 608.49.
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates closing balance ✓ correct total interest

ans only ok

- (d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur. (2 marks)

Solution
Withdrawal is interest for one quarter: $525\,000 \times 0.076 \div 4 = 9975$
Amount of withdrawal is \$9975.
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates withdrawal must be interest for one quarter ✓ correct amount

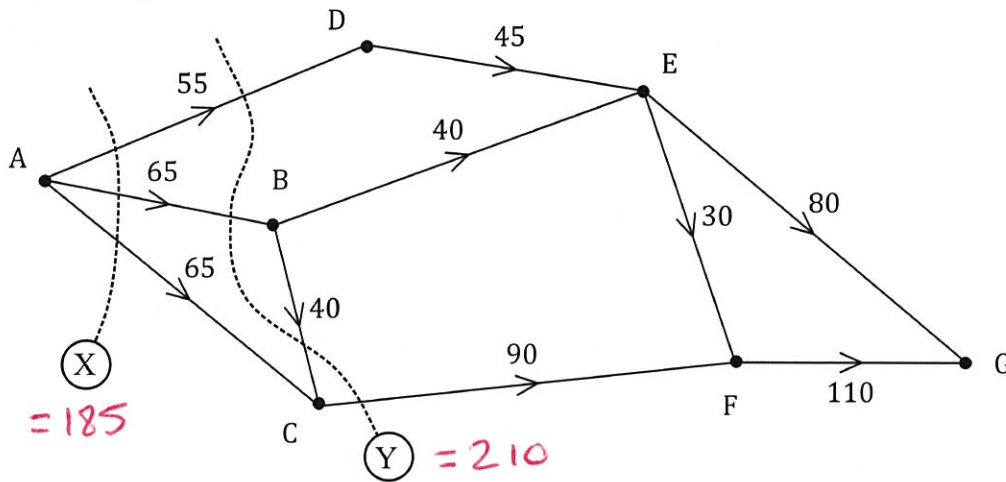
ans only ok

8

Question 19

(7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



- (a) Determine the value of cut X and the value of cut Y . (2 marks)

Solution
Cut $X = 55 + 65 + 65 = 185$ vpm. Cut $Y = 55 + 65 + 0 + 90 = 210$ vpm.
Specific behaviours
<ul style="list-style-type: none"> ✓ value of cut X ✓ value of cut Y

- (b) Determine the maximum flow of vehicles per hour from A to G . (3 marks)

Solution
$ADEG = 45$; $ABEG = 35$; $ABEFG = 5$; $ABCFG = 25$; $ACFG = 65$; Total = 175. OR Minimum cut is $45 + 40 + 90 = 175$. ✓ Maximum hourly flow is $175 \times 60 = 10\,500$ vehicles per hour. ✓ f.t from max flow
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates systematic listing or several cuts ✓ working ✓ correct maximum flow per minute ✓ correct maximum flow per hour

- (c) City engineers recommend taking steps to improve traffic flow between junctions D and E . Determine, with reasoning, the maximum increase in the hourly flow of vehicles from A to G that their plan could achieve. (2 marks)

Solution
Spare capacity along $ADEFG$ is $10 \cdot 0 \cdot 25 \cdot 15$, so increase by 10. OR Minimum cut not using DE is 185, so $185 - 175 = 10$. ✓ Maximum possible increase is $10 \times 60 = 600$ vehicles/hour. ✓
Specific behaviours
<ul style="list-style-type: none"> ✓ reasoning ✓ correct increase

ans only 1 mark

Question 20

(7 marks)

An electronics store advertises a TV for sale at a price of \$5950. Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with 24 monthly payments of \$281.20. The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

- (a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

Solution
Repay: $24 \times 281.20 = \$6748.80$ ✓
Interest: $6748.80 - 5950 = \$798.80$ ✓
Specific behaviours
✓ calculates total repaid
✓ calculates interest

ans
only
ok

- (b) Determine the annual percentage interest rate that applies to the loan. (2 marks)

Solution
Using Financial app: $N = 24,$ $PV = 5950,$ $PMT = -281.20,$ $FV = 0,$ $P/Y = 12,$ $C/Y = 12$ ✓
Solve for $I\% = 12.4$, so rate is 12.4% per annum. ✓
Specific behaviours
✓ shows correct values for financial calculator
✓ correct annual rate

ans
only
ok

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of 13.98% and over 36 months.

- (c) Determine their monthly repayment. (2 marks)

Solution
Using Financial app with: $N = 36,$ $PV = 5950,$ $I\% = 13.98,$ $FV = 0,$ $P/Y = 12,$ $C/Y = 12$ } ✓
Solve for $PMT = -203.30$, so their repayment is \$203.30 per month. ✓
Specific behaviours
✓ shows correct values for financial calculator
✓ correct repayment

ans
only
ok

- (d) Determine the total interest that the bank will charge on the loan over the 36 months. (1 mark)

Solution
Using Financial app: $\sum INT = -1368.77$. Using $36 \times 203.30 - 5950 = 1368.80$. Total of \$1368.77 or \$1368.80 in interest. ✓
Specific behaviours
✓ correct amount

See next page

Question 21

(8 marks)

For each quarter in the 11 years from 2005 to 2015, a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year 2013 is shown in the table below.

Quarter	1	2	3	4
Mean daily users	537.8	474.9	471.5	529.5
Deseasonalised mean daily users	508.6	510.1	495.4	499.5

The trend line for the deseasonalised mean daily users is $\hat{n} = 673 - 5.3t$ where t is the quarter and $t = 1$ corresponds to the first quarter of 2005.

- (a) State, with justification, whether the mean number of users per day was increasing or decreasing over the 11 years. (1 mark)

Solution
Decreasing - slope of trend line is negative ✓
Specific behaviours
✓ states decreasing, justifies with slope

- (b) Calculate the seasonal indices and enter them in the following table. (2 marks)

Quarter	1	2	3	4
Seasonal Index	1.057	0.931	0.952	1.060
Solution				
See table: Decimals to at least 2 dp (or equivalent as percentage). $537.8 \div 508.6 = 1.057, 0.931, 0.952, 1.060$				
Specific behaviours				
✓ at least two correct and indices sum to 4 ✓ ✓ all correct ✓				

- (c) State, with justification, in which quarter the library was usually least busy during the period that data was collected. (2 marks)

Solution
Second quarter - the seasonal index is lowest for this quarter. ✓
Specific behaviours
✓ correct quarter ✓ ✓ justifies using indices

- (d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the second quarter of 2015. (3 marks)

Solution
$t = 42$ ✓
$\hat{n} = 673 - 5.3(42) = 450.4$ ✓
$450.4 \times 0.931 = 419.32$
Estimate there were 419 users. ✓
Specific behaviours
✓ correct value of t ✓ calculates \hat{n} ✓ seasonalises to obtain correct estimate

ans only
2 marks

8